？JMC Logic and Reasoning Answers

(Answers for questions 1a0, 1b, and 1c are included here. The rest of the questions in the paper are the same as COMP40018 Reasoning exam, so answers for those can be found in the COMP40018 folder. If you spot any mistakes, feel free to correct them. Big thanks to Jyry for uploading your answers :D)

1a) (φ **→** ψ) ∧ ¬(¬ρ ∨ ψ) into DNF

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| (φ **→** ψ) ∧ ¬(¬ρ ∨ ψ) | | Original formula |
| ≡ | (¬φ ∨ψ) ∧ ¬(¬ρ ∨ ψ) | A **→** B ≡ ¬A ∨ B |
| ≡ | (¬φ ∨ψ) ∧ (¬¬ρ ∧ ¬ψ) | De Morgan’s Law |
| ≡ | (¬φ ∨ψ) ∧ (ρ ∧ ¬ψ) | ¬¬A ≡ A |
| ≡ | (¬φ ∧ (ρ ∧¬ψ)) ∨ (ψ ∧ (ρ ∧¬ψ)) | Distributivity of ∧ over ∨ |
| ≡ | (¬φ ∧ ρ ∧¬ψ) ∨ (ψ ∧ ρ ∧¬ψ) | Associativity of ∧ |
| ≡ | (¬φ ∧ ρ ∧¬ψ) ∨ (ψ ∧ ¬ψ ∧ρ) | A ∧ B ≡ B ∧ A |
| ≡ | (¬φ ∧ ρ ∧¬ψ) ∨ (⊥ ∧ρ) | A ∧ ¬A ≡ ⊥ |
| ≡ | (¬φ ∧ ρ ∧¬ψ) ∨ ⊥ | ⊥ ∧ A ≡ ⊥ |
| ≡ | ¬φ ∧ ρ ∧¬ψ | A ∨ ⊥ ≡ A |

1b) It is sound.

Informally, ¬∧I is just ∨I but with De Morgan’s law applied to it, i.e. ¬(¬A ∧ ¬B) ≡ A ∨ B.

We could also show it is a derived rule.

1ci) “x is the smallest value in xs” becomes:

∀ y, i : Nat . [((xs !! i) = y) **→** (x ≤ y)]

1cii) “the value x occurs at least twice in xs” becomes:

∃ i, j : Nat . [¬(i = j) ∧ ((xs !! i) = x) ∧ ((xs !! j) = x)]

1ciii) “all values in xs occur at least twice in ys” becomes:

∀ x, i : Nat . [((xs !! i) = x) **→** twice(x, ys)]

1civ) “there are exactly two values that are repeated in xs” becomes:

∃ x, y : Nat . [¬(x = y) ∧ twice(x, xs) ∧ twice(y, xs) ∧ (∀ z : Nat . [twice(z, xs) **→** (z = x) ∨ (z = y)])]